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Difficulties in learning algebra: An analysis of students' errors

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Difficulties in learning algebra: An analysis of students' errors

T I Pramesti[1], H Retnawati[2]

[1]Mathematics Education Department of Graduate School, Yogyakarta State University Sleman, D. I. Yogyakarta, Indonesia 55281
[2]Mathematics Education Department, Yogyakarta State University Sleman, D. I. Yogyakarta, Indonesia 55281

trisyliaip.2017@student.uny.ac.id

Abstract. Algebra is an important domain that first studied at the junior high school. This domain will be useful in obtaining the concept of another domain. However, learning algebra in the early grades has some challenges for students. This study aims to describe students' difficulties when learning algebra in the early grade. This study is a qualitative research with phenomenology approach. Participants in this study are 65 students from junior high schools in Bantul Regency, Province of Special Region of Yogyakarta, Indonesia. The data collected used test. Data are analyzed using Milles & Huberman Model, with reduction, data display and verification. The results of the analysis show that students' difficulties in learning algebra are (1) understanding the problem;(2) understanding meaning of variables and;(3) operation algebra. Therefore, the teachers' strategy to teach algebra material have to developed to minimize students' difficulties in learning algebra.

1. Introduction

Mathematics is the queen of all science and considered impeccable in its methods, validity, and logic [1]. Mathematics has a certain method, its undoubted validity is also related to this with its consistent nature, and putting forward this logic is also related to rationality of thinking. Mathematics cannot be separated by abstract object. Nevertheless, mathematics is also a very important science part of daily activities. Mathematics in education is a concern and important to learn. Mathematics can be learned by school children when used in real life everyday because in reality some aspects of life require mathematical skills, as a means of communication, and can be used in conveying information, and giving satisfaction when solving problems [2].

Mathematics consists of various domains, one of which is algebra. Algebra is the core topic of mathematics and in junior high school becomes important for another material [3]. The success of students in algebra material will be very influential on the next material. As mentioned in Jupri's research [3] algebra is also an instrument in obtaining other mathematical domains such as analytic geometry, calculus, and statistics. So algebra is the language of knowledge, but also a big way to higher mathematics.

Algebra material includes the concept of solving arithmetic, procedures for solving problems, the relationship between quantities, and the last is structure. However, the ability of students in the algebra domain is still relatively low. This is indicated by the achievement of TIMSS results in 2007 and 2011 in the 8th grade algebra domain. Following are the TIMSS results in 2007 and 2011 in table 1.

Table 1. TIMSS results in 2007 and 2011

Years	**2007**	**2011**
International average	500	500
Math Score	427	386
Algebra Domain Scores	405	392
Rating	36 from 49	38 from 42

Other data also show that students' abilities in the algebra domain are still relatively low. This can be seen from mastery level in the algebra material in Bantul Regency, as follows in table 2.

Table 2. Mastery level in the Algebra Material in Bantul Regency Middle School

Year 2014/2015	**Year 2015/2016**	**Year 2016/2017**	**Year 2017/2018**
58.63	56.79	57.02	57.54

From the table above is known that the ability of students in 2014/2015 algebra material is 58.63%, in 2015/2016 it is 56.79% and in 2016/2017 it is 57.02%, and in 2017/2018 amounting to 57.54%. The data shows that the absorption of algebra material has decreased and increased. However, students' ability to understand algebra material in Bantul Regency is still in the low category.

Algebra learning in junior high school students is also important because students at this level just know what algebra is. Judging from the psychology of students, junior high school students in the early grades also have characteristics which are 13-16 years old. Other important characteristics are related to their cognitive abilities which are at the stage of formal operational development. At this stage students are able to reason by using relationships between objects in life to be associated with mathematical problems. So students are still in the transition stage from concrete thinking to abstract thinking. While the algebraic thinking position has been at the stage of abstract thinking. So that most students experience difficulties in learning algebra.

Many studies mention the difficulties of students in learning algebra in the process of mathematization everyday sentences to mathematical forms, understanding algebraic equations, arithmetic operations, understanding the meaning of symbols equal to and meaning of variables [3][4]. Activity mathematization same with Abstraction involves making math in symbols [5]. Activity of understanding algebraic equations and arithmetic operations includes in transformational activities. Transformational activities include procedural activities. Suppose loading factoring, substituting, simplifying expiration [6]. The principal of the transformation activity is in order to balance.

Same as Jupri's research, other research in algebra sub-material straight-line equation also shows students' errors in understanding the concept and procedural errors [7]. The lack of understanding concept also become one of types of student error in mathematical symbols, graphs and problem solving [8][9].

Other studies show students' difficulties in solving the problem of one of the algebraic linear equations of two variables in the form of reading errors, comprehension errors, transformations, skill processes and encoding [10]. Another errors in resolving HOTS problems is in the form of a number skill, a skill arithmetic process or relating to calculations, understanding information or information skills, understanding language or language skills and difficulties in understanding representations or visual-spatial skills[11]. This error also include procedures being forgotten [8]. In general, algebra becomes a problem for students [1].

Teachers need to understand students' learning difficulties in order to creat effective teaching strategies and to produce meaningful learning among students. After the teachers find the difficult indicators, they might create new learning strategies that will be meaningful for providing the students' conceptual understanding towards the students regarding the difficult indicators. Multiple learning strategies might be applied in the learning process by adjusting the students' conditions, the materials or the indicators that will be studied and the drawbacks of each student [9].

Student have low ability in this algebra material shows students have difficulties in learning algebra. Difficulties in learning algebra can be seen from how students work on algebra problems. students 'difficulties in learning algebra need to be minimized by a strategy, and first find out the students' difficulties. As Result, there should be a study that explain what difficulties are experienced when students learn algebra in Junior High School Students.

2. Method

2.1 Research Type.
This research is a qualitative study in the phenomenological approach. This research aims to explore the kesulitan when students doing algebra's task in junior high school students.

2.2 Research Participants
The research participants are 65 students of Junior High School in Bantul Regency, Province of Special Region of Yogyakarta, Indonesia. Participants have characteristics 13-15 years old. All students have studied algebra.

2.3 Instrument and Procedure
The instrument used in this study is a test question consisting of 4 items topic about algebra.

2.4 Data Analysis
Analysis of the data used is Milles & Huberman Model, with reduction, data display and verification. In the analysis of the first model data collection is document colected from students answer sheet. Second, Data reduction is a form of sharpening analysis, classify, direct, discard unnecessary data organize data in such a way that conclusions final can be drawn and verified. Data reduction takes place continuously continues as long as the research has not ended. Products from Data reduction is in the form of summaries of field notes, both from initial note, expansion, or addition.

Third, Data presentation is a series of information organizations allowing research conclusions to be made. Data presentation intended to find patterns that are meaningful as well gives the possibility of conclusions and withdrawals provide action. States that data presentation in the form of sentence narratives, images/schemes, networks and tables as the narrative. Last, Withdrawal conclusions are part of the activities complete configuration. Conclusions also verified during the study. Conclusions have been drawn since researchers conducted records, patterns, statements, configurations, causal directions, and various propositions.

3. Result
Data analysis shows some of the difficulties faced by students in working on algebraic problems. Each will be explained in each item. The percentage of students who answered correctly is shown in table 3.

Table 3. The percentage of students answer correctly

Number	Indicator Problem	The percentage of students answer correctly
1 a	Understand the problem	36.92%
1 b	Understand the meaning of variables	18.84%
2	Operate algebraic forms	53.30%
3	Solving word problems related to algebra	41.53%
4	Make algebra problems	23.07%

The percentage of students who answered correctly on question item 1a is 36.92% and 1b is 18.84%. To solve this problem, students must understand the problem to make a mathematical form.

In addition, students must understand the concept of variable meaning also. Question item 1 is shown in figure 1.

In terms of how students understand the problem, there are still many students' does not understand the problem, namely 41 students. From this problem, it is discovered that a tailor bought five logs for sewing for the uniform of the order of Maju Jaya Middle school. After all the uniforms are finished stitching, it turned out that the fabric is still 7 meters away. So that the mathematical form of the fabric used is $5x-7$, -7 shows the rest of the fabric after use. There are 24 students who correctly answer the question. One of the students' correct is shown in figure 2.

Where as from the results of the completion of the students who solve the problem correctly consider the $5x+7$ mathematical form. This means that students do not understand the problem given, that the important sentence is the remainder which means -7, not + 7.

Students who answer incorrectly state that the algebraic form is correct with the reason that 5x is the size of the stitch and 7 is the remaining stitch. Students do not pay attention to the plus minus sign and the meaning of plus minus sign. One of students' error is shown in figure 3.

A tailor bought five logs for sewing clothes for uniforms ordered by Maju Jaya Middle school. After all the uniforms are finished stitching, it turned out that the fabric was still 7 meters away. The algebraic form used to sew the uniform is $5x + 7$.

a.   Is the algebraic form true? Give your explanation.
b.   The meaning of variable x is fabric spindle. Is meaning of the variable true? Give your
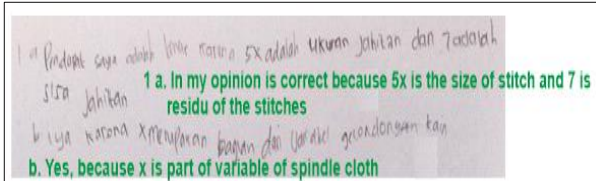
Figure 1. Question item 1

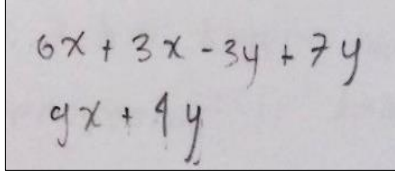**Figure 2.** One of the students who correctly answer

Figure 3. One of the students' error answer

The percentage of students who answered correctly on question item 2 is 53.30%. Question item 2 is shown in figure 4. This problem is a form of operating problem simplifying the algebraic form. In solving this problem requires procedural capability that is in operation paying attention to the same variables. Where in this case 6x is operating in addition to 3x. Then, $-3y$ is operated with $7y$. So the result is $9x + 4y$. One of the students' correct is shown in figure 5.

So that when students find it difficult to solve the problem, the procedural ability of students is still low. Most students are correct in solving it but there are still some students who are wrong in doing it. One of the students' errors is shown in figure 6.

The simple form of $6x - 3y + 3x + 7y$ is ....

Figure 4. Question item 2

Figure 5. One of the students who correctly answer	**Figure 6.** One of the students' error answer

The percentage of students who answered correctly on question item 3 is 41.53%. Question item 3 is shown in figure 7. This problem is a form of problem solving everyday problems related to algebra. In solving these problems requires the ability to understand the problem, as well as procedural abilities in applying algebraic forms. to answer this question students must know the concept of the area of a rectangle, namely by multiplying the length and width of the first, however there is important information that there is a width of the road that is equal to 1 meter. Thus, the length of the original garden $(x + 4)$ m becomes $(x + 4 - 1)$ m or equal to $(x + 3)$ m. For a fixed garden width of x meters. Then in determining the area using algebraic multiplication operations, as follows:

$$L = p \times l$$
$$L = (x + 3). x$$
$$L = x. x + 3. x$$
$$L = x^2 + 3x$$

So that the area of the park (shaded) is obtained, namely $(x^2 + 3x)$ m². One of the students' correct is shown in figure 8.

Students who answered incorrectly, did not fully understand the problem given, most students ignored the information on the width of the road, so the width of the park remained $(x + 4)$ m. One of the students' errors is shown in figure 9.

The picture below shows a rectangular garden, the white area is a road with a width of 1 meter. State the area of the park (shaded) in m².
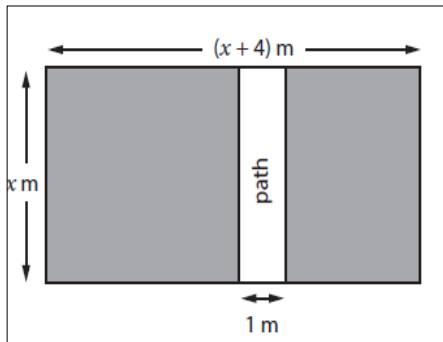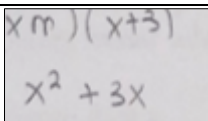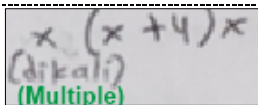

Figure 7. Question item 3

Figure 8. One of the students who correctly answer	**Figure 9.** One of the students' error answer

The percentage of students who answered correctly on question item 4 is 8.3%. Question item 4 is shown in figure 10. This problem is a form of reasoning problem that students are required to create. As well as the presence of creative thinking are required in the matter. The ability to understand the meaning of variables is also seen. One of the students' correct is shown in figure 11.

Some students cannot solve the problem, because they are not familiar with the type problem. So the role of the teacher in developing creative thinking skills. One of the students' errors is shown in figure 12.

Make a problem stating the following algebraic form along with the meaning of the variable:
   a. $7x - 11 = 24$
   b. $5x + 7y + 2z$

Figure 10. Question item 4

Figure 11. One of the students who correctly answer

Figure 12. One of the students' error answer

4. Discussion

The students' error in preparing the questions related to algebra include at least three difficulties, namely understanding the problem, understanding the meaning of variables, and operating the algebraic form. Every error or difficulty has a different factor. These factors must be minimized.

The first students' errors is understanding the problem. Understanding the problem in problem solving ability is an important matter that needs to be considered [11]. In understanding the problem is also one form of error that many students do, stated in the study by Wijaya's research [12] students experienced errors in understanding the PISA problem, namely the process of creating a mathematical model. This is due to several factors, one of which is the error of reading and understanding information [10]. Other causes of difficulties in mathematics problems of students in problem solving skills, namely difficulties in understanding visual aspects and spatial, as well as difficulties in doing perception (spatial)[13].

Not absorbing information properly is one of the factors that influence student errors [7]. This error is seen when students make algebraic expressions. The process of making algebraic expressions is called mathematization, which is also a form of student difficulties in learning algebra [3]. The last

that related to this problem is student not understand with creativity problem. This kind of problem is HOTS problem. Therefore, building HOTS-oriented learning can be conducted through minimalizing eacher domination and maximizing the role of students in the learning process [9][14].

The second students' errors is in understanding the meaning of variables. This error is included in a misunderstanding of the concept. In this case related to algebra material, students' conceptual misconception is on the concept of variable meaning. This is in line with research into the basic conceptual errors of algebra [13]. Misconceptions on certain material will be very influential with previous concepts and influence other related concepts [7][15].

The third error is in operating the algebraic form. this error will also be related to procedural capabilities. Procedural errors are operating rules in algebraic form. This error is also related to the accuracy of students working on it.

5. Conclusion

The students' error in preparing the questions related to algebra include at least three things, namely understanding the problem, understanding the meaning of variables, and operating the algebraic form. Various advanced studies can be carried out in relation to the difficulties that students do in learning algebra. Therefore, the teachers' strategy to teach algebra learning material can be developed so as to minimize student difficulties in learning algebra.

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